Table 1 Endurance test results

Test average impulse bit	27.5 μ lb-sec
Total propellant used	9.74 in. or 0.888 lb
Test average I_{sp}	405 sec
Total impulse delivered in this test	359 lb-sec
Anticipated total impulse capability	387 lb-sec

ance data. Furthermore, operation of the thrusters in conjunction with a spacecraft type telemetry system has been carried out without any consequence. For this reason, the elimination of this noise has had a low priority.

After initial thrust measurements, thruster Unit B was subjected to the SMS qualification level vibration and acceleration tests. This consisted of a sinusoidal vibration schedule in each of the three principal axes, a random vibration schedule in each axis, and two steady-state acceleration tests. After each of these individual tests, power was applied to the thruster at atmospheric pressure to verify proper voltage levels and igniter sequencing. No problems of any kind were encountered. During the sinusoidal tests a stroboscope was used to look for any distortions or resonance that might occur. None could be seen. After the series of vibration and acceleration tests were completed, thruster Unit B was retested for thrust and electrical performance. The values obtained during these tests agreed with those taken prior to vibration.

Performance of the thruster through the scheduled $-40°C$ and $-20°C$ thermal-vacuum tests was without incidence. Instrumentation revealed that the thruster electrical performance was excellent throughout the entire range of duty cyles, input levels, and firing rates. However, on the last operational check prior to increasing the temperature to $+50°C$ a failure occurred. Upon disassembly of the thruster it was found that one of the two energy storage capacitors was shorted. As a result of the low temperature environment, radial cracks developed in the hard epoxy end seals on the capacitor. This permitted air, entrapped between the layers of foil and mylar during manufacture, to slowly be pumped out. Thruster operation continued normally with no indication of a fault until the pressure was reduced to a critical value and a Paschen breakdown occurred within the capacitor.

The immediate problem was solved by replacing the hard epoxy end-seals with a flexible polyurethane potting compound that was compatible with the capacitor material and temperatures. After replacing the capacitors the thermal-vacuum test was begun anew. This time the thruster performed satisfactorily under all combinations of temperatures and input signal variations without anomalous deviations. Perhaps the most important observations made are that the fully charged voltage (1450 v) of the 2-μf energy storage capacitors remaining essentially constant ($\pm 0.2\%$) under all test conditions; and the capacitor temperature did not exceed 150°F.

During additional tests, the thruster was operated for 2×10^6 pulses at 100 ppm and a 90% duty cycle to simulate its spacecraft usage. This was accomplished by energizing the thruster "Enable" input at the desired duty cycle while leaving the "Input Power" and the "Fire Command" inputs on continuously. The number of on/off cycles exceeded what would be required for daily use in a 5-yr mission.

After thrust measurements, the test was continued. All electrical performance data remained very stable and indicated normal operation. After 2.7 million pulses, however, a failure occurred within the thruster. Instead of completely shutting down during the "Off" period the high-voltage converter portion of the thruster remained on and kept the capacitors charged. When fully on, however, thruster performance was not affected. Because of this and the fact that the required number of on/off cycles had already been exceeded, the test was continued. "Normal" thruster system operation could be realized under these conditions merely by applying or removing the 29.4-v input power in lieu of the enable signal. Thruster operation was resumed and continued to 13.1 million pulses at which time the test was terminated to ensure that sufficient propellant would be available for additional thrust measurements. Throughout the test, all telemetry and voltage monitors remained normal and very stable.

Thrust measurements carried out after the endurance test yielded an impulse bit of 27.7 μ lb-sec. Previous work indicates that for a given configuration the product of $I_{bit} \times I_{sp}$ is constant. This factor in conjunction with the propellant consumption rate, which was measured throughout the endurance test, was used to calculate the test average impulse bit, as shown in Table 1. This value was then used to calculate the test average I_{sp} and total impulse. The anticipated total impulse capability shown in Table 1 was based on the propellant used before the endurance test and the available propellant at completion.
The results of Table 1 are in excellent agreement with earlier results obtained at Fairchild Republic.

Conclusions and Recommendations

1) The thrusters structural design is sound and is capable of satisfying the SMS qualification vibration and acceleration levels.

2) No thermal problems were encountered with the basic thruster design. The capacitor failure is considered a component problem in which more developmental work is recommended.

3) The basic thrust and electrical performance was very good but additional effort is needed to locate the source of and to eliminate the noise spikes on the telemetry lines.

4) A failure analysis of the "Enable" circuit has not yet been carried out so no recommendations can be made as to how it might be fixed.

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Optimal Scheduling of a Satellite Control Network Using Zero-One Linear Programing

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I. Introduction

AT any given time this country has a large number of satellites in Earth orbit. These are sponsored by a number of agencies for purposes ranging from weather observation to defense and hence their characteristics and orbits vary widely.

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The sponsors necessarily require occasional communication with their satellites to receive data or transmit commands, but to provide flexibility at reasonable cost they must do this communication using shared networks of ground stations. Because the opportunities to communicate with a given satellite are limited to the intervals of time in which the satellite can maintain radio contact with one of the ground stations, and because each ground station can process only one or two satellites at a time, the scheduling of these Satellite Control Networks (SCN) to meet the sponsors' needs is a difficult and time-consuming task. This Note describes a new approach to optimal automated scheduling of an SCN.

The unique character of the SCN scheduling problem arises because many important satellites, particularly those in low orbits, are potentially contactable only a few times a day for a few minutes at a time. Hence the fundamental scheduling question may be phrased, "Given that a certain satellite is visible to the network at a certain time, should the resources of the SCN be used to contact it then?" This question must be answered for many satellites and several ground stations in such a way that the sponsors' communications needs and the network constraints are satisfied. Previous attempts at automatic solution of this problem have been confined to special cases,[1] have been heuristic,[2] or have involved enumeration of problem subsets.[3] In this Note, zero-one linear programing (0-1 LP), an optimization technique discussed, for example, by Wagner[4] and used for other, dissimilar scheduling problems in Refs. 5–7, is proposed as the basic solution tool.

II. Use of Zero-One Linear Programing

Zero-one linear programing (0-1 LP) was chosen for finding the optimal allocation of a given set of SCN resources to servicing a set of passes. Other techniques were considered but rejected. Of these, the most important were dynamic programing, for which a suitable problem formulation could not be found, and Monte Carlo and heuristic approximation methods, which did not guarantee than an optimum would be found. This section describes the application of 0-1 LP to the SCN problem.

The 0-1 LP problem is to find a vector χ with components χ_i, $i = 1, i = 1, 2, \ldots, n$, which minimizes the scalar function

$$J = \sum_{i=1}^{n} C_i \chi_i \tag{1}$$

where $C_i \geqq 0$ for $i = 1, \ldots, n$, subject to the m constraints

$$b_j + \sum_{i=1}^{n} a_{ji} \chi_i \geqq 0 \qquad j = 1, 2, \ldots, m \tag{2}$$

and the special constraints

$$\chi_i = 0 \quad \text{or} \quad 1 \qquad i = 1, \ldots, n \tag{3}$$

There exist several algorithms for computing solutions to such problems.

The formulation of the SCN problem for 0-1 LP requires four basic steps which will be elaborated upon below.

1) Assign to each specific independently schedulable activity a variable χ_i, and make the interpretations that $\chi_i = 0$ implies that the activity, with all of its associated resource utilizations, is to be performed and that $\chi_i = 1$ implies the contrary.

2) Associate with χ_i a numerical value C_i for the activity which χ_i represents.

3) By examining the resource utilization implications of all variables, determine inequality constraints which will ensure the viability of the final 0-1 LP allocation.

4) Solve the 0-1 LP problem created by 1–3.

The third step is clearly the crucial one, and several examples are given to demonstrate that an SCN's equipment limitations can indeed be represented in the form required by the 0-1 LP format.

A. Variable Definition

The variables χ_i, which in the operation of the algorithms take on the values zero and one, are interpreted as representing the logical decisions "yes, perform the action represented by this variable" and "no, do not perform the action." The variables are ordinarily defined to represent the servicing of a particular pass, i.e., satellite visibility at a ground station, in a precisely defined manner, and presumably any significant alternative manner of servicing the same pass, such as an extra remote station turnaround-time allowance or a different antenna location, is represented by a different variable.

For example, consider a pass visible to remote station C from 1330 to 1338, needing 30 min of prepass station setup (turnaround time, or TAT), and requiring a resource set consisting of the entire ground station, its data lines to a central station, and portions of the central station equipment. Let the entire "single thread" of individual resources be treated as a single set called Resource C, and assume that prepass and postpass activities are not required. Then variable χ_5 might represent: "Should Resource C be used from 1300 until 1338 to service the pass by satellite 1234 at station C?" The time interval allows for the preferred TAT to be contiguous to the 8 min pass. Variable χ_6 might then be assigned to represent the same situation, but with times from 1320 to 1338, i.e., with minimum TAT. More complicated resource requirements can, of course, be represented, and variables can also be used to represent activities such as preventive maintenance.

B. Cost Function

Associated with each variable χ_i is a utility C_i which may be thought of either as the payoff for performing the ith action or as the cost of not performing it. It is assumed that such a value should exist, since the variable does represent a pass or other worthy activity. Establishing the numerical values is difficult, however, and leads into utility theory.

C. Constraints

Once the variables χ_i and values C_i are defined, it is seen that an allocation which minimizes the cost function (1) where n is the number of variables, will represent an optimum utilization of the system provided that the resources are not overburdened and the allocation is not redundant in the sense of, for example, having the effect of servicing the same pass twice with different TAT's.

Inequality constraints of the form (2) are used to insure that the finite resources are not overburdened. Thus the formation of constraints is critical if a feasible problem solution is to be obtained. Since the 0-1 variables have some of the characteristics of logical variables, it is not surprising that the constraints often can be used to reflect logical decisions also.

The principles of constraint generation are simple ones. In order not to obscure them with notation, they are presented below *via* examples. The generalization of the examples should be apparent.

The most common constraints are due to resource limitations. These are generated by examining each resource in turn and finding the total demand on that resource by the activities represented by the variables for each time interval of interest. If the demand exceeds the available resource capability then it follows that several activities must remain undone, i.e., that the variables representing some of the activities must be forced to take on the value 1 by a suitable constraint. Suppose, for example, that satellites 1234, 2345, and 3456 will be visible simultaneously to ground station C, which has only one tracking antenna. Then at most one of them can be contacted. If these passes have been assigned variables χ_5 and χ_6, χ_{14}, and χ_{47}, respectively, then

$$\chi_5 + \chi_6 + \chi_{14} + \chi_{47} \geqq 3$$

will enforce this constraint. If servicing these passes would require 15, 25, and 45 min of computer time at the station between 1200 and 1300 and if servicing a pass by satellite 4567 represented by variable χ_3 would require 22 min of computer time in the same interval, a further constraint is necessary. Since 107 min would be needed in a 60 min interval, passes representing at least 47 min, and hence variables representing at least 62 min, of potential demand must remain unserviced. The constraint

$$22\chi_3+15(\chi_5+\chi_6)+25\chi_{14}+45\chi_{47}\geqq 62$$

will clearly cause this.

Another common constraint arises because two or more variables may in fact represent alternative allocations which would satisfy a single need. Satellite 1234 has already been assigned variables χ_5 and χ_6 at station C. It may be that a pass an hour later at station A would also satisfy the sponsor's communication need and that this pass has been assigned variable χ_{251}. Since multiple contacts would be wasteful of the SCN resources and the satellite's energy capacity, a constraint is introduced to ensure that at most one of the alternatives is used. This means at least two must remain unused, or that

$$\chi_5+\chi_6+\chi_{251}\geqq 2$$

Many other system constraints and logical situations can be represented within the 0-1 LP, paradigm, including various logical implications. A user quickly becomes familiar with the possibilities once he learns the basic ones above. Care must be exercised, however, to avoid creating contradictory constraints which require a variable to simultaneously take on the values zero and one. This is best avoided by using constraints only to keep resources from being overburdened and not to force passes to be serviced.

III. Preliminary Resource Allocation Program

In order to experiment with the above approach, a limited resource allocation program called RAP was programed to run on a CDC 6600 computer. A fully operational allocation program reflecting the entire potential of 0-1 LP will take a large effort and considerable further research to develop. The limited version, nevertheless, provides useful insight into the nature of the SCN problem, and for this reason its properties are considered in the following paragraphs.

The computational flow of RAP is very simple and is essentially that of Fig. 1. A Geoffrion algorithm[8] is used for the 0-1 LP optimization. RAP is a study program rather than a production program, but certain of its characteristics are worth examining.

1) The program is written entirely in FORTRAN for CDC 6600/7600 computers. It requires about 3500 cards, and the instructions use about 6000 words of memory. No auxiliary storage is used. The program fills core for large problems unless the matrices, which tend to be sparse, are packed.

2) Running time depends upon both problem size and the character of the constraints. Many problems have been solved using only a single pass through the standard simplex algorithm utilized by the Geoffrion 0-1 LP algorithm. In such cases scenarios involving 500 passes yielding 800 0-1 variables have been processed in less than two minutes of CPU time.

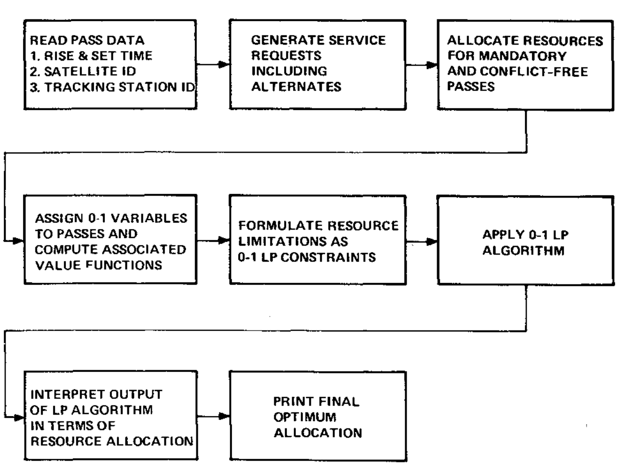

Fig. 1 Processing flow of RAP program for SCN resource allocation.

3) It has been difficult to compare the automated schedule directly with that produced by a human. A small scenario processed by RAP proved impossible for a human to improve. In real scheduling, however, human schedulers are allowed to bend rules which the computer presently treats as inviolate.

The limited experience achieved so far with this program along with discussions with potential users of such a system have indicated that the method has significant potential for performing system studies and as an aid in planning real schedules. Its ability to always produce an optimum within its ground rules will lend credibility to such tradeoff studies as the cost/benefits of deleting or improving resources and will remove much tedious detail from real scheduling exercises.

The experience and discussions have also revealed that advanced versions of RAP would need certain improvements. Larger scenarios must be handled, and decisions concerning scheduling of communications with high-altitude satellites, which have long visibility intervals, can probably be made more efficiently than by RAP's method of discretizing the intervals into a small number for which "yes-no" decisions are made. Furthermore, the previous comment (3) indicates that the program should either be modified to do some "rule-bending" or it should be run in an interactive mode.

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Effect of Transverse Products of Inertia on Re-Entry Vehicle Trim Angle Behavior

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Nomenclature

c.g. = center of gravity
C_A = axial force coefficient

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